## Range and Outliers

## Focus on...

After this lesson, you will be able to...determine the range for data setsidentify outliers in data sets


The wooden roller coaster at Playland in Vancouver was built in 1958. It is one of the oldest wooden roller coasters that is still in use. Most newer roller coasters are made of steel.

## Discuss the Math

## How can the largest and smallest values be used to describe a set of data?

The following diagram shows the elevation changes for the Stomach Bender roller coaster. The measurements represent the heights of the roller coaster relative to the starting point of the ride.


1. Copy the table below into your notebook.

| Location Along Ride | Start | A | B | C | D | E | End |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Elevation Relative <br> to Starting Point (m) |  |  |  |  |  |  |  |

2. Fill in the elevations at each point.
3. a) What is the largest value?
b) What is the smallest value?
c) What is the range of heights for the roller coaster?
4. Do you think there are any outliers in the set of 7 locations on the roller coaster? If so, what are they?

## Reflect on Your Findings

5. How would you use the highest and lowest values of a data set to help you describe the data?

## Example 1: Calculate the Range

A hospital recorded the following number of births one week.

## range

- the positive difference between the largest and smallest values in a data set


## outlier

- a value that is much smaller or larger than the other data values
- a data set may have one or more outliers or no outliers

| Day | Mon | Tues | Wed | Thurs | Fri | Sat | Sun |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Births | 7 | 10 | 4 | 6 | 7 | 8 | 3 |

a) What is the highest number of births?
b) What is the lowest number of births?
c) What is the range of the data?

## Solution

a) The highest number of births is 10 births on Tuesday.
b) The lowest number of births is 3 births on Sunday.
c) The range is calculated by subtracting the lowest value from the highest value.

$$
\begin{aligned}
\text { Range } & =\text { highest value }- \text { lowest value } \\
& =10-3 \\
& =7
\end{aligned}
$$



## Show Youknow

What is the range of each set of values?
a) $7,4,3,6,2,5,8,3$
b) $11,8,8,5,7,6,7,6,6$
c) $-4,3,-1,0,2$

## Example 2: Identify Outliers in a Data Set

In the 1985-1986 NHL hockey season, Wayne Gretzky set a record for the most points (goals + assists) recorded during a single regular season. The following table shows the top five point leaders for the 1985-1986 season and the 2005-2006 season.

| NHL Point Leaders |  |  |  |
| :--- | :---: | :--- | :---: |
| 1985-1986 Season | Points | 2005-2006 Season | Points |
| Wayne Gretzky | 215 | Joe Thornton | 125 |
| Mario Lemieux | 141 | Jaromir Jagr | 123 |
| Paul Coffey | 138 | Alexander Ovechkin | 106 |
| Jari Kurri | 131 | Dany Heatley | 103 |
| Mike Bossy | 123 | Daniel Alfredsson | 103 |

a) Determine the range for each season. Compare the two ranges.
b) Which data value appears to be an outlier in one of the two seasons?
c) Remove the outlier and determine the new range for that season. Compare the new range with the range from the other season.

## Solution

a) 1985-1986 Season:

215 is the highest value, and 123 is the lowest value
Range $=215-123$

$$
=92
$$

2005-2006 Season:
125 is the highest value, and 103 is the lowest value
Range $=125-103$

$$
=22
$$

The ranges are very different: 92 and 22 .
b) The outlier appears to be Gretzky's 215 points in the 1985-1986 season.
c) The new highest value is 141 .

The new range $=141-123$

$$
=18
$$

The two ranges are more comparable in size: 22 and 18.

## Show You Know

What value(s) appear to be outliers in each set of data?
a) $18,19,79,17,20,12$
b) $14,14,16,15,1,16$
c) $98,202,99,91,2,95,89,88,94$

## Rey ldeas

- The range provides information about the spread of the data.

Range $=$ highest value - lowest value

- Outliers are values that are very different from the rest of the numbers in a set of data. A data set can have no outliers, or one or more outliers.


## Communicate the Ideas

1. In your own words, explain the meaning of range and outliers.
2. How can you determine the smallest value in a data set if you are given the range and the largest value? Use an example to explain your response.

## Practise

For help with \#3 to \#5, refer to Example 1 on page 435.
3. What is the range of each set of data?
a) $11,8,7,10,6,5,18,13$
b) $4,-5,3,-2,0,7$
4. Determine the range of each set of data.
a) $16,11,7,29,31,18,21,18$
b) $7,1,-1,-2,9,-7,4$
5. Leanne timed how long she could hold her breath while swimming under water. She recorded the following timings, in seconds: $47,55,42,65,61$
What is the range of the data?

For help with \#6 to \#8, refer to Example 2 on page 436.
6. What value(s) appear to be outliers in each set of data?
a) $6,9,9,37,8,7$
b) $24,34,46,26,32,43,115$
c) $48,32,67,61,47,95,89,888,1$
7. Identify any possible outlier(s) in each set of data.
a) $666,11,9,12,8,13,10$
b) $43,54,62,64,0,211,45$
c) $82,75,76,85,89,95,92,88,80$
8. Curb-cut ramps are often built at intersections to allow wheelchairs and strollers to cross streets more easily. In one town, a study was done to count the number of curb-cuts at ten random intersections. The results for five different neighbourhoods are given:
$9,8,8,1,7$
a) What is the range?
b) Which data value may be an outlier?
c) If you remove the outlier, what is the new range?


## Apply

9. Vincent received a 200-piece jigsaw puzzle for his birthday. The following times indicate how long it took him to complete the puzzle the first six times he did it:
$54 \mathrm{~min}, 36 \mathrm{~min}, 34 \mathrm{~min}, 29 \mathrm{~min}$, $27 \mathrm{~min}, 28 \mathrm{~min}$
a) What is the range?
b) Which time may be an outlier?
c) Why might this particular value be so different from the others?
d) If you remove the outlier, what is the new range?
10. The following table shows the mean high temperature for each month in Whitehorse, Yukon Territory.

| Month | Mean Temp. |
| :--- | :---: |
| January | $-13^{\circ} \mathrm{C}$ |
| February | $-7^{\circ} \mathrm{C}$ |
| March | $-1^{\circ} \mathrm{C}$ |
| April | $6^{\circ} \mathrm{C}$ |
| May | $13^{\circ} \mathrm{C}$ |
| June | $18^{\circ} \mathrm{C}$ |
| July | $20^{\circ} \mathrm{C}$ |
| August | $18^{\circ} \mathrm{C}$ |
| September | $12^{\circ} \mathrm{C}$ |
| October | $4^{\circ} \mathrm{C}$ |
| November | $-5^{\circ} \mathrm{C}$ |
| December | $-11^{\circ} \mathrm{C}$ |

a) What is the highest value?
b) What is the lowest value?
c) What is the range of the data?
11. The table gives the mean number of wet days per year for several locations across Canada. What is the range of wet days?

| Location | Mean Number of <br> Wet Days |
| :--- | :---: |
| St. John's | 217 |
| Halifax | 170 |
| Fredericton | 156 |
| Toronto | 139 |
| Winnipeg | 119 |
| Regina | 109 |
| Edmonton | 123 |
| Vancouver | 164 |
| Whitehorse | 122 |
| Yellowknife | 118 |

12. The table gives the magnitudes of five of the largest earthquakes that have occurred in western Canada.

| Location | Date | Magnitude |
| :--- | :---: | :---: |
| West of <br> Vancouver Island, <br> BC | Jan 26, 1700 | 9.0 |
| South of Hope, <br> BC | Dec 14, 1872 | 7.4 |
| West Coast of <br> Vancouver Island | Dec 6, 1918 | 7.0 |
| Central <br> Vancouver Island | June 23, 1946 | 7.3 |
| Queen Charlotte <br> Islands, BC | Aug 22, 1949 | 8.1 |

Magnitude refers to the relative size of an earthquake. Magnitude measurements are recorded using the Richter scale, which is a special scale that goes from 0 to 10 . What is the range of magnitudes of the five earthquakes in the table?

## WWW Web Link

To learn more about earthquakes in Canada, go to www.mathlinks7.ca and follow the links.
13. Final times, in minutes, for the top ten contestants in a race were 55.2, 56.7, 56.9, 57.3, 58.8, 58.9, 59.2, 59.3, 59.3, and 59.7.
a) What is the range of times?
b) Do you think this set of data contains any outliers? Explain.

## Extend

14. A data set contains five numbers:

12, 20, 18, 15, 5
a) Determine the range, median, mode, and mean of the data set.
b) Add 10 to each of the five numbers and recalculate the range, median, mode, and mean. How do your answers compare with your answers to part a)?
c) Multiply the numbers in the original data set by 10 and calculate the new measures of central tendency. How do your answers compare with your answers to part a)?
15. Five students competed in a pie-eating contest.

- The range of times for the group to finish eating their pies was 9 min.
- The least amount of time it took for someone to finish their pie was 1 min .
- The mode was 4 min .
- One boy took 2 min more than the median.
a) What are the finishing times for the five contestants?
b) What was the mean number of minutes it took the five contestants to eat their pies?
c) What strategy did you use to determine the mean?

16. Conduct the following experiment to simulate rolling two 10 -sided dice each numbered from 0 to 9 .

- Randomly choose a page from your textbook.
- Calculate the sum of the last two digits of the page number. For example, for page 108 you would add 0 and 8 to get 8 .
- Randomly choose 49 more pages for a total of 50 pages.
- Copy and complete the frequency table.

| Sum | Tally | Freq. | Sum | Tally | Freq. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | 10 |  |  |
| 1 |  |  | 11 |  |  |
| 2 |  |  | 12 |  |  |
| 3 |  |  | 13 |  |  |
| 4 |  |  | 14 |  |  |
| 5 |  |  | 15 |  |  |
| 6 |  |  | 16 |  |  |
| 7 |  |  | 17 |  |  |
| 8 |  |  | 18 |  |  |
| 9 |  |  | 19 |  |  |

a) What is the mode?
b) What is the mean?
c) What is the median?
d) What is the range?
e) Why are there no tally marks on 19?

## MATH LINK

Measure and record the heights of ten people in your class, including your teacher.
a) What is the range of heights?
b) Identify any possible outliers.

