Median and Mode
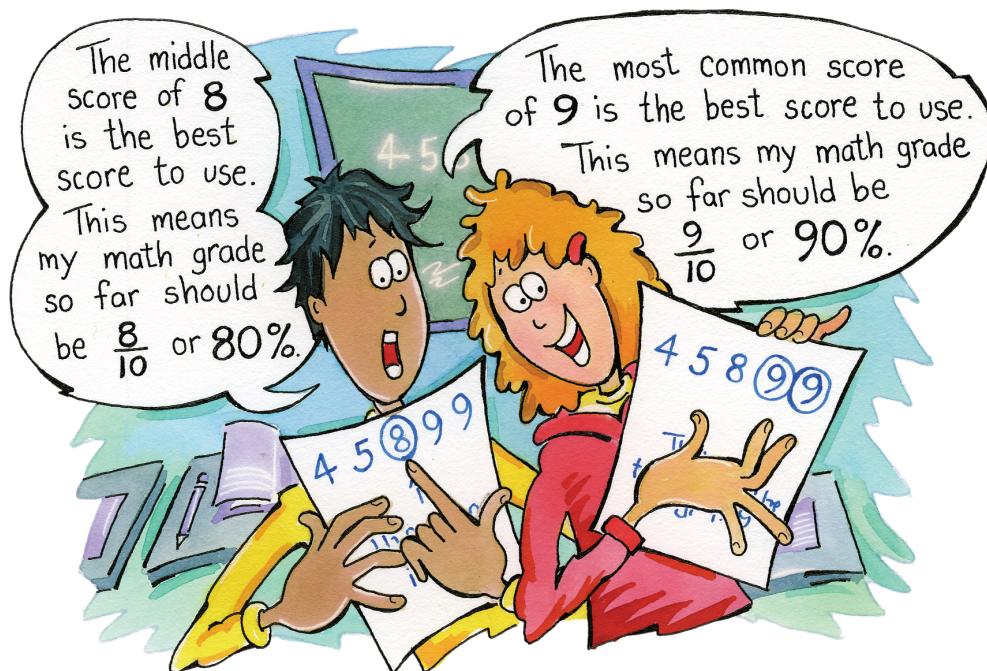
Early in the school year, Melanie and Amir had the exact same scores on five weekly math quizzes. Their scores out of 10 were 8, 9, 4, 5, 9. Their teacher asked them to report on a single score that best summarizes their progress in math.
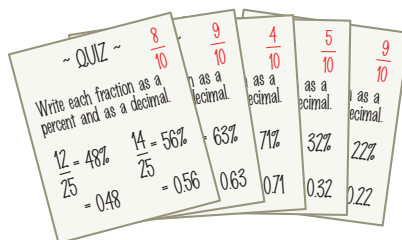
Discuss the Math

How do you determine median and mode?

1. Which student do you think is right? Discuss your response with a classmate. Do you agree with your classmate?

2. Both Amir and Melanie have justified their decisions. They have each found a ▮measure of central tendency▮ to represent the "centre" of the data.

   a) Amir has calculated the ▮median▮ of the five scores. Describe how he found the median.

   b) Melanie has calculated the ▮mode▮ of the five scores. Describe how she found the mode.

Reflect on Your Findings

3. a) Which measure, median or mode, do you think better represents how these students are doing in math this term? Explain why.

   b) Give a set of ten numbers where you think the median is a more accurate measure of central tendency.

   c) Give a set of ten numbers where you think the mode is a more accurate measure of central tendency.

Example 1: Determine the Mode and the Median

One month, the Edmonton Oilers won 7 out of 10 games.

As a team they scored the following number of goals in these 10 games:

5, 4, 3, 2, 1, 5, 3, 3, 7, 3

   a) What is the mode for the number of goals scored?

   b) What is the median number of goals scored?

Solution

   a) Arrange the numbers in increasing order.
The mode is the most frequently occurring number in the list.
1 2 3 3 3 3 4 5 5 7
The mode is 3 since it occurs four times.

   b) Arrange the numbers in increasing order. The median is the middle value. Cross off pairs of least and greatest values until you reach the middle one or two numbers.
1̶ 2̶ 3̶ 3̶ 3 3 4̶ 5̶ 5̶ 7̶
If one number remains, then it is the median value.
If two numbers remain then you must find the value halfway between them. In this case, since the two numbers are both 3, the median is 3.
Notice that both the mode and median are 3 in this example. The mode and median will sometimes be the same value, but not always.

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measure of central tendency

- a value that represents the centre of a set of data
- can be the mean, median, or mode

median

- the middle number in a set of data after the data have been arranged in order
- median of 2, 5, 6, 8, and 9 is 6
- median of 1, 3, 6, 8, 9, and 10 is 7

mode

- the most frequently occurring number in a set of data
- mode of 3, 5, 7, 7, and 9 is 7
- modes of 2, 2, 4, 6, 6, 8, and 11 are 2 and 6
- the data 1, 3, 4, and 5 has no mode

Example 2: Determine the Mode and Median From a Frequency Table

The frequency table shows the hourly wages of employees at a recycling depot.

Hourly Wages ($)	Number of Employees
8	3
10	2
11	3
14	2

a) What is the mode for the hourly wages?

b) What is the median wage?

c) How would the mode and median be affected if one employee gets a raise from $10 per hour to $11 per hour?

Solution

a) *Method 1: List the Values in Order*

Record the hourly wages, in dollars, for each employee in increasing order.

8 8 8 10 10 11 11 11 14 14

There are three 8s and three 11s.

So, there are two modes: $8 and $11.

> Since there are three people who earn $8 per hour, record three 8s.

> If no number is repeated in a set of data then there is no mode.

Method 2: Use the Frequency Table

From the frequency table, you can determine the mode by looking at the numbers in the second column. The highest number is 3. It corresponds to 3 people who earn $8 and 3 people who earn $11. So, there are two modes: $8 and $11.

> **Literacy Link**
>
> When there are two modes the data are said to be *bimodal*. The prefix "bi" means two. For example, a bicycle has two wheels.

b) *Method 1: List the Values in Order*

Record the hourly wages, in dollars, for each employee in increasing order. The median is the middle value.

Cross off pairs of least and greatest values until you reach the middle one or two numbers.

8̶ 8̶ 8̶ 1̶0̶ 10 11 1̶1̶ 1̶1̶ 1̶4̶ 1̶4̶

Since two numbers remain, find the middle value between 10 and 11.

> To calculate the middle value, you could add the numbers and then divide by two.
> $$\frac{10 + 11}{2} = \frac{21}{2}$$
> $$= 10.5$$

> The median does not have to be a number in a set of data.

The median hourly wage is $10.50.

Method 2: Use the Frequency Table

Since there are ten employees, the median must be halfway between the wages of the fifth and sixth employees when the salaries are arranged in order. From the frequency table, five people earn $8 or $10 and five people earn $11 or $14.

The median must be halfway between $10 and $11.

The median hourly wage is $10.50.

c) By changing one $10 to $11 the ordered values would now be

8   8   8   10   11   11   11   11   14   14

The only mode is $11, since $11 now occurs four times.

The fifth number is $11 and sixth number is $11, so the median is $11.

Show You Know

Find the mode and median prices of the baseball caps sold in the last week.

Baseball Cap Price ($)	Number of Sales
7	5
9	5
10	6
12	4

Key Ideas

- The mode is the most frequently occurring number in a set of data.
  - If no number repeats, there is no mode.
    1   2   3   4   5   No mode
  - There can be more than one mode.
    1   1   2   2   3   Two modes: 1 and 2 (bimodal)
- The median is the middle value in a set of data after the numbers have been arranged in order.
  $\cancel{1}$ $\cancel{2}$ 3 $\cancel{4}$ $\cancel{5}$ Median is 3
- If there is an even number of data values, then the median is the value halfway between the two middle numbers.
  $\cancel{2}$ $\cancel{4}$ 6   8 $\cancel{10}$ $\cancel{12}$

  The median here is the value halfway between 6 and 8, or 7.

  $$\frac{6+8}{2} = \frac{14}{2} = 7$$

- The median does not have to be a number in the set of data.

Communicate the Ideas

1. Over ten days, the following numbers of juice cans were collected by the school recycling team: 15, 20, 12, 16, 24, 20, 12, 20, 23, 17.
   Which value do you think is easier to determine, mode or median? Why?

2. Create a set of five numbers where the median and mode are the same. Explain why you chose the numbers you did.

3. Dana was asked to find the median of the following numbers: 3, 6, 5, 4, 2. She decided to order the numbers from greatest to least: 6, 5, 4, 3, 2. She removed outer pairs of numbers until she was left with the middle value, 4. Has she correctly found the median? Explain.

Practise

4. What are the median and mode of each set of data?
   a) 4, 2, 9, 6, 4
   b) 21, 15, 18, 21, 20, 18
   c) 3, 8, 5, 12, 10, 8, 2

5. Determine the median and mode of each set of data.
   a) 6, 4, 8, 6, 2, 9
   b) 14, 5, 8, 11, 10
   c) 18, 24, 16, 18, 24, 16, 18, 18

6. In one week, a store in the mall sold the following numbers of Nickelback CDs:
   34, 42, 37, 44, 46, 42, 51
   What were the mode and median for the CD sales that week?

7. In 12 hockey games, the Calgary Flames scored the following numbers of goals:
   4, 4, 0, 2, 3, 3, 4, 1, 3, 0, 3, 2
   What were the mode and median?

8. A new T-shirt company sold 26 shirts on their first day. The table shows the number of T-shirts sold

Price ($)	Number Sold
8	6
14	7
17	5
20	4
25	4

   according to price. What were the mode and median for the T-shirt prices?

9. A coffee shop sold 36 beverages one hour. The prices of the beverages sold are shown in the

Price	Number Sold
$2	12
$3	10
$3.50	9
$4	5

   table. What were the mode and median prices?

10. A cat gives birth to a litter of six kittens. The masses of the kittens are:

95 g, 100 g, 100 g, 105 g, 110 g, 110 g

a) What is the mode?

b) What is the median mass?

c) The mother cat has a second litter. The masses of these kittens are:

90 g, 95 g, 100 g, 105 g, 110 g, 115 g

What are the median and mode of the masses of all 12 kittens?

11. The following tally chart shows the heights of a grade 7 boys' basketball team.

Height (cm)	Number of Players
150	||
155	|
160	||
165	||||
170	⊞
180	|

a) What is the mode for the heights? How can you determine the mode(s) quickly from the chart?

b) What is the median height?

12. What is one possible set of four numbers that have a mode of 7 and a median of 11?

Extend

13. Five whole numbers have a mode of 4 and a median of 3. What are the five numbers?

14. If the only mode is 4, describe all possible whole number values for n.

a) 5, 6, 7, 4, 2, n

b) 5, 6, 4, 4, n

15. If the median is 4, describe all possible whole number values for n.

a) 3, 2, 6, 7, n

b) 1, 3, 4, 5, n

16. The median of the set of numbers 3, 4, 5, x, and y is 5. What are the possible whole number values for x and y?

17. Five classmates have birthdays in March. The median of their birth dates is 12 (March 12) and the mode is 8. If the sum of the five birth dates is 56, find all possible sets of birth dates.

MATH LINK

David surveyed ten friends about their shoe size. He recorded the following sizes:

6, 7, 5, 8, 8, 7, 7, 6, 9, 8

a) What is the median shoe size?

b) What is the mode?