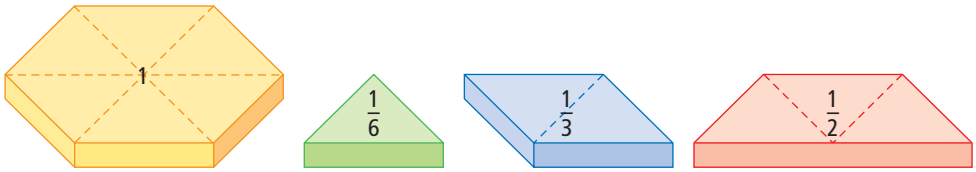
7.2 Add and Subtract Fractions With Unlike Denominators



Focus on...

After this lesson, you will be able to...

- add and subtract fractions with unlike denominators
- solve problems involving the addition and subtraction of fractions
- check that your answers are reasonable using estimation

How could you use pattern blocks to model addition and subtraction?

Explore the Math

How can you add and subtract fractions with unlike denominators?

Materials

- pattern blocks
- coloured pencils

1. a) What two pattern blocks would you use to represent $\frac{1}{2}$ and $\frac{1}{3}$?

b) Can you tell what the answer to $\frac{1}{2} + \frac{1}{3}$ is using these two pattern blocks? Explain.

2. a) Use the green triangles to represent $\frac{1}{2}$ and $\frac{1}{3}$. What fraction does each green triangle represent?

b) Can you tell what the answer to $\frac{1}{2} + \frac{1}{3}$ is now? Explain.

3. a) What pattern blocks would you use to represent $\frac{1}{2}$ and $\frac{1}{6}$?

b) Can you tell what the answer to $\frac{1}{2} - \frac{1}{6}$ is using these two pattern blocks? Explain.

4. a) Use the green triangles to represent $\frac{1}{2}$.

b) Can you tell what the answer to $\frac{1}{2} - \frac{1}{6}$ is now? Explain.

c) How many green triangles are left?

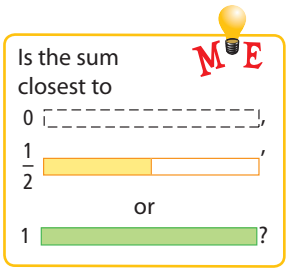
Reflect on Your Findings

5. How can you use pattern blocks to help you add and subtract fractions with unlike denominators?

Example 1: Add Fractions With Unlike Denominators

Add. Write the answer in lowest terms.

$\frac{1}{3} + \frac{1}{6}$

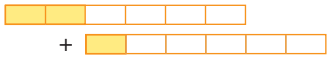


Solution

Method 1: Use Fraction Strips

$\frac{1}{3} + \frac{1}{6}$

To add, you need to use parts that are the same size.

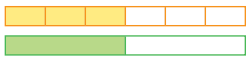


Each part represents $\frac{1}{6}$. Count the parts.

$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$

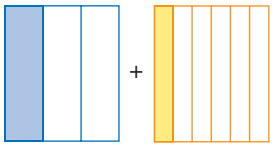
Write the answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$



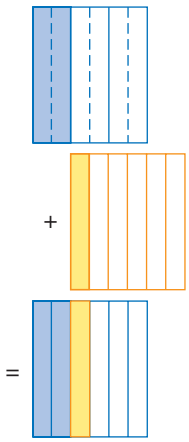
Method 2: Draw a Diagram

$\frac{1}{3} + \frac{1}{6}$



To add, you need to use parts that are the same size.

$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$



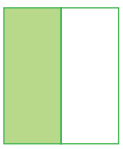
Strategies

Solve a Simpler Problem

Refer to page xvii.

Write the answer in lowest terms.

$\frac{3}{6} = \frac{1}{2}$



Method 3: Use a Common Denominator

The denominator of $\frac{1}{3}$ is 3.

Multiples of 3 are 3, (6), 9, 12, …

The denominator of $\frac{1}{6}$ is 6.

Multiples of 6 are (6), 12, 18, 24, …

The first multiple divisible by both 3 and 6 is 6.

A common denominator is 6.

Write equivalent fractions with 6 as the denominator.

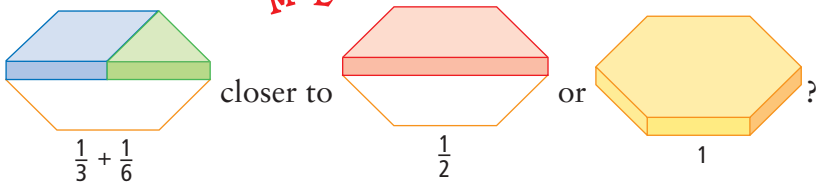
$$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$$
$$= \frac{2 + 1}{6} \quad \text{Add the numerators.}$$
$$= \frac{3}{6}$$

Write the answer in lowest terms.

$$\frac{3}{6} \overset{\div 3}{=} \frac{1}{2}$$

Check:

Use pattern blocks.



It equals $\frac{1}{2}$. The answer is reasonable.

Show You Know

Add. Write each answer in lowest terms.

a) $\frac{1}{6} + \frac{2}{3}$ **b)** $\frac{1}{10} + \frac{4}{5}$ **c)** $\frac{1}{2} + \frac{4}{8}$

WWW Web Link

To learn more about adding fractions, go to www.mathlinks7.ca and follow the links.

Literacy Link

When the numerator equals the denominator, the fraction is equal to 1.

$\frac{3}{3} = 1$ $\frac{16}{16} = 1$

Example 2: Subtract Fractions With Unlike Denominators

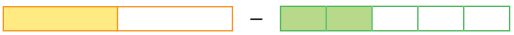
Subtract.

$\frac{1}{2} - \frac{2}{5}$

Solution

Method 1: Use Fraction Strips

$\frac{1}{2} - \frac{2}{5}$



To subtract $\frac{2}{5}$, you need parts that are the same size.

Subtract.

$\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Check:

Is $\frac{1}{10}$ closer to 0 or $\frac{1}{2}$?

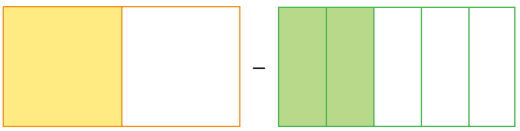
$0 = \frac{0}{10} \quad \frac{1}{2} = \frac{5}{10}$

$\frac{1}{10}$ is a little more than $\frac{0}{10}$, or 0.

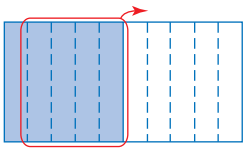
Compare this to the estimate you made before you subtracted.

Method 2: Draw a Diagram

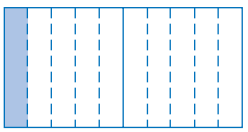
$\frac{1}{2} - \frac{2}{5}$



To subtract $\frac{2}{5}$, you need parts that are the same size.



Subtract.



$\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Method 3: Use a Common Denominator

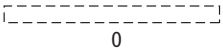
The denominator of $\frac{1}{2}$ is 2.

Multiples of 2 are 2, 4, 6, 8, (10), ...

The denominator of $\frac{2}{5}$ is 5.

Multiples of 5 are 5, (10), 15, 20, ...

Is the difference closer to



0

or

$\frac{1}{2}$

The first multiple divisible by both 2 and 5 is 10.

A common denominator is 10.

Write equivalent fractions with 10 as the denominator.

$$\frac{1}{2} - \frac{2}{5} = \frac{5}{10} - \frac{4}{10}$$
$$= \frac{5 - 4}{10} \quad \text{Subtract the numerators.}$$
$$= \frac{1}{10}$$

Show You Know

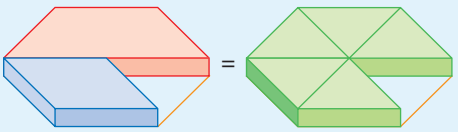
Subtract. Write each answer in lowest terms.

a) $\frac{3}{4} - \frac{1}{2}$ **b)** $\frac{2}{3} - \frac{1}{4}$ **c)** $\frac{3}{4} - \frac{1}{8}$

Key Ideas

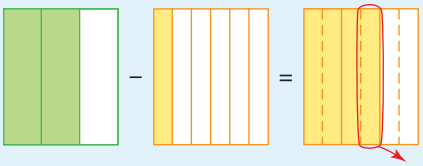
- When adding and subtracting fractions using models or diagrams, show each fraction using parts of the whole that are of equal size.

Pattern Blocks



$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$

Diagram



$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6}$$

- To add or subtract fractions with unlike denominators, use a common denominator.
- You can estimate when adding or subtracting fractions by comparing fractions to 0, $\frac{1}{2}$, or 1.

Communicate the Ideas

1. How are $\frac{1}{3}$ and $\frac{2}{6}$ alike? How are they different?

2. a) How would you use diagrams to calculate $\frac{1}{4} + \frac{1}{2}$?

b) Compare your answer with a classmate's.

3. Why is it difficult to calculate $\frac{1}{2} - \frac{1}{8}$ without changing $\frac{1}{2}$ to $\frac{4}{8}$?

Practise

For help with #4 to #7, refer to Example 1 on pages 238–239.

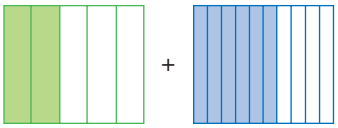
4. Write each addition statement shown by the fraction strips. Estimate and then add.

a) 

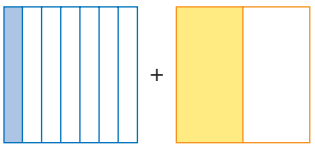
b) 

c) 

5. For each diagram, write an addition statement. Then add.

a) 

b) 

c) 

6. Add. Write your answers in lowest terms.

a) $\frac{2}{5} + \frac{1}{10}$

b) $\frac{5}{8} + \frac{1}{4}$

c) $\frac{1}{3} + \frac{5}{12}$

d) $\frac{1}{4} + \frac{3}{5}$

e) $\frac{1}{2} + \frac{1}{5}$

f) $\frac{3}{8} + \frac{1}{6}$

7. Determine the sum. Write your answers in lowest terms.

a) $\frac{1}{2} + \frac{3}{8}$

b) $\frac{1}{12} + \frac{5}{6}$

c) $\frac{2}{10} + \frac{4}{5}$

d) $\frac{1}{3} + \frac{2}{9}$

e) $\frac{2}{5} + \frac{1}{2}$

f) $\frac{1}{6} + \frac{3}{4}$

8. Write each addition statement shown by the pattern blocks. Then add.

a) 

b) 

For help with #9 to #12, refer to Example 2 on pages 240–241.

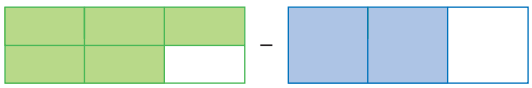
9. Write each subtraction statement shown by the fraction strips. Estimate and then subtract.

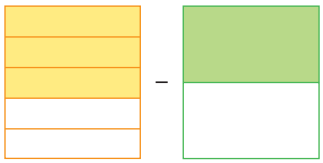
a) 

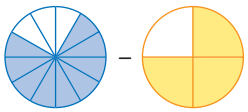
b) 

c) 

10. For each diagram, write a subtraction statement. Then subtract.

a) 

b) 

c) 

11. Subtract. Write your answers in lowest terms.

a) $\frac{3}{5} - \frac{3}{10}$

b) $\frac{5}{6} - \frac{1}{2}$

c) $\frac{1}{2} - \frac{1}{10}$

d) $\frac{7}{8} - \frac{1}{2}$

e) $\frac{2}{3} - \frac{2}{5}$

f) $\frac{5}{8} - \frac{5}{12}$

12. Determine the difference. Write your answers in lowest terms.

a) $\frac{3}{4} - \frac{1}{8}$ **b)** $\frac{11}{12} - \frac{5}{6}$

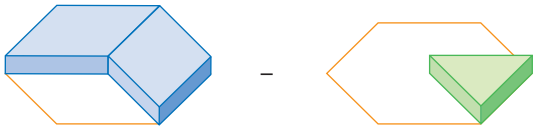
c) $\frac{2}{3} - \frac{1}{2}$ **d)** $\frac{1}{6} - \frac{1}{9}$

e) $\frac{2}{5} - \frac{1}{4}$ **f)** $\frac{5}{6} - \frac{11}{15}$

13. Write each subtraction statement shown by the pattern blocks. Then subtract.

a)

b)



Apply

14. The students made muffins in cooking class. They get to take some muffins home. There are 12 muffins in a muffin tray.

a) John says, "I'm taking $\frac{1}{4}$ of a tray."

Katie says, "I'm taking $\frac{1}{3}$ of a tray."

What fraction of a tray are John and Katie taking altogether?

b) Marjoe says, "I'm taking $\frac{1}{6}$ of a tray."

Sandeep says, "I'm taking $\frac{1}{12}$ of a tray."

What fraction of a tray are Marjoe and Sandeep taking altogether?

15. Zach was leading in a swimming race by $\frac{5}{8}$ of a length. He won the race by $\frac{1}{2}$ a length. By how much did the second-place swimmer catch up by the end of the race?

16. A friend shows you the following work for an addition problem.

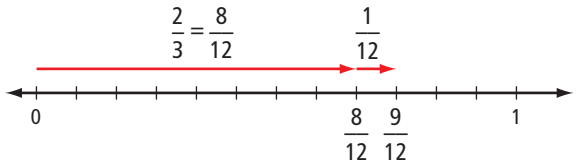
$\frac{1}{4} + \frac{1}{3} = \frac{2}{7}$

a) Explain the error in your friend's work.

b) Use a diagram to show the correct answer.

17. An airplane was loading in Pond Inlet for its flight to Iqaluit, Nunavut. The plane was $\frac{1}{6}$ full of passengers and $\frac{1}{3}$ full of cargo. How much space was left?

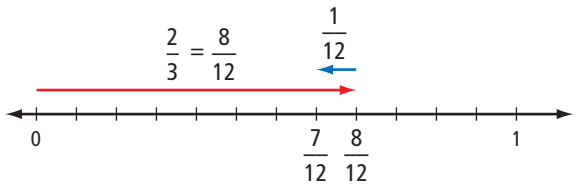
18. You can use a number line to show $\frac{2}{3} + \frac{1}{12} = \frac{9}{12}$.



Draw number lines to add the fractions.

a) $\frac{1}{4} + \frac{1}{4}$ **b)** $\frac{1}{2} + \frac{1}{8}$ **c)** $\frac{3}{10} + \frac{3}{5}$

19. You can use a number line to show $\frac{2}{3} - \frac{1}{12} = \frac{7}{12}$.

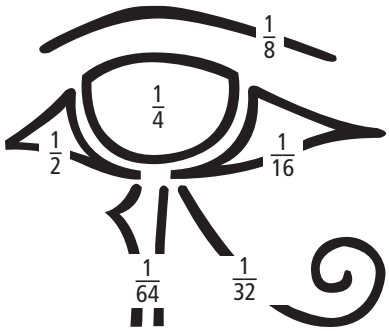


Draw number lines to subtract the fractions.

a) $\frac{1}{2} - \frac{1}{8}$ **b)** $\frac{1}{4} - \frac{1}{12}$ **c)** $\frac{5}{6} - \frac{1}{4}$

Extend

20. The ancient Egyptians thought the fractions in the Eye of Horus added up to 1. Were they correct? Show your work.



21. Water is pumped into a pool. After one hour, $\frac{1}{5}$ of the pool is filled.

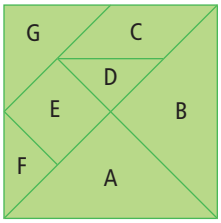
a) After 3 h, how full is the pool?

b) How long does it take in total to fill the pool?

22. The sum of each row, column, and diagonal in this magic square must equal 1. Copy the square and fill in the blanks.

■	■	$\frac{5}{12}$
$\frac{7}{12}$	$\frac{1}{3}$	■
$\frac{1}{4}$	■	■

23. A tangram is a square puzzle that is divided into seven shapes.



a) Suppose piece A is $\frac{1}{4}$. What are the values of pieces B, C, D, E, F, and G?

b) What is the sum of A and B?

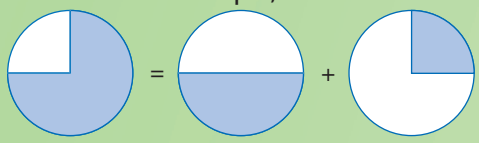
c) Subtract the value of D from the whole.

d) Which two tangram pieces add up to the value of C?

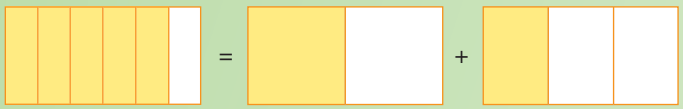
e) Make a problem of your own using tangram pieces. Have a classmate solve it.

MATH LINK

The Egyptians of 3000 B.C.E. used only unit fractions. These are fractions with a numerator of 1, such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. They wrote all other fractions as sums of unit fractions. For example,



$$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$$



$$\frac{5}{6}=\frac{1}{2}+\frac{1}{3}$$

These sums are called Egyptian fractions.

a) Add the unit fractions. Use diagrams to show your work.

$\frac{1}{4}+\frac{1}{8}$ $\frac{1}{3}+\frac{1}{9}$

b) Which one of the two sums in a) is greater? By how much?

c) How would ancient Egyptians have written $\frac{5}{12}$ as the sum of two unit fractions?