7.1 Common Denominators

Focus on...

After this lesson, you will be able to...

- find a common denominator for a set of fractions
- compare and order positive fractions

Jasmin and Tyler collect trading cards. Jasmin has collected $\frac{1}{3}$ of a set. Tyler has collected $\frac{1}{4}$ of a set. They want to know who has more cards. Jasmin and Tyler need to compare the fractions. It is easier to compare fractions when the denominators are the same. So, Jasmin and Tyler need to find a common denominator.

Explore the Math

Materials

- coloured pencils

How can you determine a common denominator?

1. Fold a piece of paper into 3 equal parts. Shade $\frac{1}{3}$ of the paper red.

2. Fold the same piece of paper into 4 equal parts the other way.

3. **a)** How many equal parts is the paper divided into?
   b) Count how many parts you shaded red. Name an equivalent fraction for $\frac{1}{3}$ using your answer to part a) as the denominator.

4. Fold a different piece of paper into 4 equal parts. Shade $\frac{1}{4}$ of the paper blue.

5. Fold the piece of paper into 3 equal parts the other way.

6. Count how many parts you shaded blue. Name an equivalent fraction for $\frac{1}{4}$.

Reflect on Your Findings

7. **a)** What is the relationship between the denominators 3 and 4, and the denominator 12?
   b) What is one method for determining a **common denominator**?

common denominator

- a common multiple of the denominators of a set of fractions
- a common denominator for $\frac{1}{4}$ and $\frac{1}{6}$ is 12 because a common multiple of 4 and 6 is 12

Example: Determine a Common Denominator

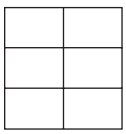
a) Determine a common denominator for $\frac{2}{3}$ and $\frac{1}{2}$.

b) Determine equivalent fractions for $\frac{2}{3}$ and $\frac{1}{2}$ using the common denominator from a).

Solution

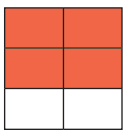
Method 1: Use Paper Folding or Diagrams

a) Divide a rectangle into 3 equal parts. Either fold a piece of paper, or draw a rectangle.
Fold the paper or divide the rectangle into 2 equal parts the other way.
There are 6 parts in the rectangle.



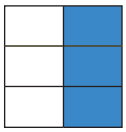
A common denominator for $\frac{2}{3}$ and $\frac{1}{2}$ is 6.

b) Shade $\frac{2}{3}$ of the rectangle red.



4 of the 6 parts are red. $\frac{2}{3} = \frac{4}{6}$

Turn the paper over, or draw another rectangle and divide it as in step a).
Shade $\frac{1}{2}$ of this rectangle blue.



3 of the 6 parts are blue. $\frac{1}{2} = \frac{3}{6}$

multiple

- the product of a given number and a natural number like 1, 2, 3, and so on
- for example, some multiples of 3 are 3, 6, 9, 12, and 15

Method 2: Use Multiples

a) The denominator of $\frac{1}{2}$ is 2.

Multiples of 2 are 2, 4, (6), 8, 10, 12, …

The denominator of $\frac{2}{3}$ is 3.

Multiples of 3 are 3, (6), 9, 12, 15, …

The first multiple divisible by both 2 and 3 is 6.

A common denominator is 6.

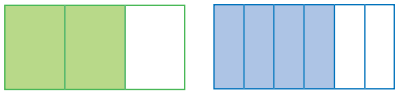
You can use divisibility rules to find multiples.
6 is divisible by both 2 and 3.
So, multiples of both 2 and 3 will be multiples of 6:
6, 12, 18, …

You could use any multiple of 6 as the common denominator, but the first multiple is often better to use. The denominator will be a smaller number, which is easier to work with.

b) Write equivalent fractions using 6 as the denominator.

$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6}$ $\quad$ $\frac{2}{3} \overset{\times 2}{=} \frac{4}{6}$

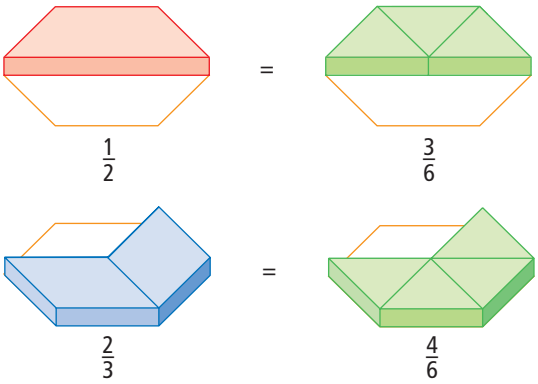
To determine equivalent fractions, multiply the numerator and denominator by the same number. This process does not change the value of the fraction.



$\frac{2}{3} = \frac{4}{6}$

Check:

Use pattern blocks.



Strategies

Model It

Refer to page xvi.

Show You Know

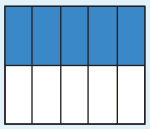
Determine a common denominator for each pair of fractions. Then use the common denominator to write equivalent fractions. Show two different methods.

a) $\frac{1}{3}$ and $\frac{3}{4}$ $\quad$ **b)** $\frac{5}{8}$ and $\frac{1}{6}$

Key Ideas

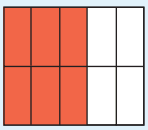
- You can use paper folding, diagrams, or multiples to determine a common denominator.

Paper Folding or Diagrams



5 of the 10 parts are blue.

$\frac{1}{2} = \frac{5}{10}$



6 of the 10 parts are red.

$\frac{3}{5} = \frac{6}{10}$

Multiples

The denominator of $\frac{1}{2}$ is 2. Multiples of 2 are 2, 4, 6, 8, (10), …

The denominator of $\frac{3}{5}$ is 5. Multiples of 5 are 5, (10), 15, 20, …

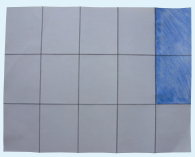
A common denominator is 10.

- To write fractions with a common denominator, determine equivalent fractions.

$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$ $\quad$ $\frac{3}{5} \overset{\times 2}{=} \frac{6}{10}$

Communicate the Ideas

1. Tina wanted to find a common denominator and equivalent fractions for $\frac{3}{5}$ and $\frac{2}{3}$. This is what she did:



a) Was she correct? If not, what was her error?

b) Draw diagrams to show what she should have done.

c) Discuss your diagrams with a classmate.

2. Ian says, "A common denominator for $\frac{3}{4}$ and $\frac{5}{6}$ is 12." Meko says, "I think it is 10." Do you agree with Ian or Meko? Why?

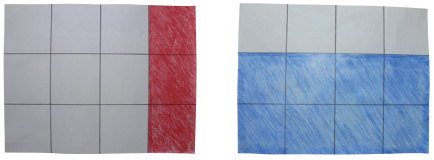
3. How can you use multiples to find a common denominator for the fractions $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{4}$?

Practise

For help with #4 to #9, refer to the Example on pages 231–232.

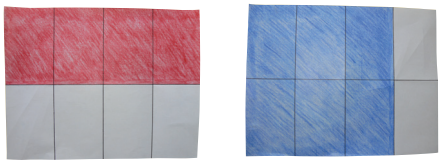
4. Use the folded papers shown to determine a common denominator and equivalent fractions for each pair of fractions.

a)



$\frac{1}{4}$ $\frac{2}{3}$

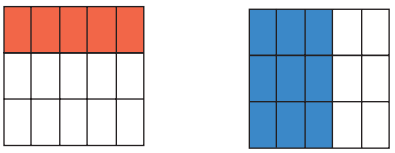
b)



$\frac{1}{2}$ $\frac{3}{4}$

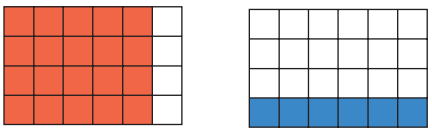
5. Look at the diagrams to determine a common denominator and equivalent fractions for each pair of fractions.

a)



$\frac{1}{3}$ $\frac{3}{5}$

b)



$\frac{5}{6}$ $\frac{1}{4}$

6. Draw a diagram to determine a common denominator for each pair of fractions. Then use the common denominator to write equivalent fractions.

a) $\frac{1}{2}$ and $\frac{1}{3}$ **b)** $\frac{2}{3}$ and $\frac{1}{5}$ **c)** $\frac{1}{6}$ and $\frac{2}{5}$

7. Use a diagram to determine a common denominator for each pair of fractions. Then write equivalent fractions using the common denominator.

a) $\frac{3}{8}$ and $\frac{1}{3}$ **b)** $\frac{5}{6}$ and $\frac{3}{4}$ **c)** $\frac{1}{5}$ and $\frac{1}{2}$

8. Use multiples to determine a common denominator for each set of fractions. Then write equivalent fractions using the common denominator.

a) $\frac{1}{2}$ and $\frac{2}{5}$ **b)** $\frac{1}{3}$ and $\frac{1}{4}$ **c)** $\frac{5}{8}$, $\frac{1}{6}$, and $\frac{5}{12}$

9. Using multiples, determine a common denominator for each set of fractions. Then use the common denominator to write equivalent fractions.

a) $\frac{3}{8}$ and $\frac{1}{4}$ **b)** $\frac{1}{6}$ and $\frac{1}{4}$ **c)** $\frac{1}{5}$, $\frac{2}{3}$, and $\frac{7}{10}$

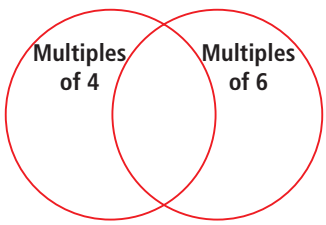
Apply

10. Determine a common denominator for each pair of fractions. Which is the larger fraction in each pair?

a) $\frac{3}{4}$, $\frac{13}{16}$ **b)** $\frac{5}{7}$, $\frac{36}{49}$

c) $\frac{11}{30}$, $\frac{3}{10}$ **d)** $\frac{12}{27}$, $\frac{4}{9}$

11. Draw a Venn diagram like the one shown to list common denominators that are less than 50 for $\frac{1}{4}$ and $\frac{1}{6}$.



12. Fill in the blanks to make equivalent fractions.

a) $\frac{1}{4} = \frac{\blacksquare}{8} = \frac{\blacksquare}{12} = \frac{\blacksquare}{16} = \frac{\blacksquare}{20} = \frac{\blacksquare}{24} = \frac{\blacksquare}{28}$

b) $\frac{1}{5} = \frac{2}{\blacksquare} = \frac{3}{\blacksquare} = \frac{4}{\blacksquare} = \frac{5}{\blacksquare} = \frac{7}{\blacksquare} = \frac{11}{\blacksquare}$

c) $\frac{24}{56} = \frac{12}{\blacksquare} = \frac{6}{\blacksquare} = \frac{3}{\blacksquare} = \frac{48}{\blacksquare} = \frac{9}{\blacksquare}$

d) $\frac{30}{48} = \frac{15}{\blacksquare} = \frac{10}{\blacksquare} = \frac{5}{\blacksquare} = \frac{\blacksquare}{96} = \frac{\blacksquare}{32}$

13. Fill in each blank with a numerator to make the statement true. Provide as many answers as possible. Use diagrams to show how you determined your answers.

a) $\frac{1}{4} < \frac{\blacksquare}{2} < \frac{3}{4}$

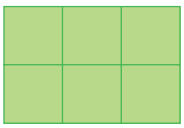
b) $\frac{1}{3} < \frac{\blacksquare}{6} < \frac{5}{6}$

c) $\frac{2}{5} < \frac{\blacksquare}{10} < \frac{4}{5}$

14. Determine a common denominator for the set of fractions. Use the common denominator to write an equivalent fraction for each fraction. Then list the fractions in order from least to greatest.

$\frac{1}{3}, \frac{1}{4}, \frac{5}{6}, \frac{2}{3}, \frac{3}{4}, \frac{1}{2}$

15. The ancient Greeks thought of numbers as being represented by rectangles. They would have made a rectangle like this to represent 6:



a) How could this rectangle be used to find a common denominator for $\frac{1}{2}$ and $\frac{1}{3}$? Explain.

b) Use a rectangle to find a common denominator for $\frac{3}{4}$ and $\frac{1}{7}$.

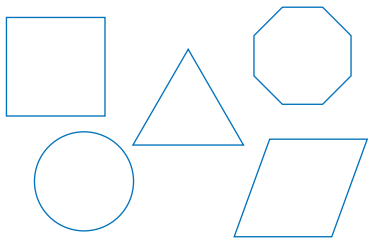
16. $\frac{5}{12}$ of a schoolyard is taken up by grass. $\frac{7}{18}$ is the track. The rest is pavement.

a) What common denominator could be used to compare these fractions?

b) Does the grass or the track take up more space?

Extend

17. a) Copy the shapes. For each shape, colour in $\frac{3}{8}$.



b) Which shapes were more difficult to colour in? Which were easier? Explain.

c) Imagine you are using paper folding to determine a common denominator for $\frac{3}{8}$ and $\frac{2}{5}$. Which of the shapes would it be possible for you to use? Show the work by drawing the fold lines on the shapes.

d) Compare your drawings with a classmate's.

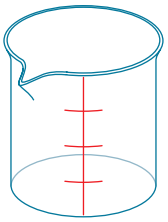
18. Write as many different proper fractions in lowest terms as you can that have denominators from 2 to 9 and numerators that are positive numbers.

19. Which of the following fractions is closest to $\frac{3}{10}$?

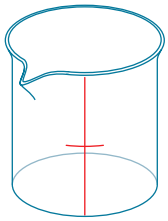
A $\frac{1}{4}$ **B** $\frac{21}{100}$ **C** $\frac{9}{40}$ **D** $\frac{2}{5}$

20. You have three beakers that are the same size. $\frac{2}{3}$ of beaker 1 contains oil. $\frac{1}{4}$ of beaker 2 contains water. Beaker 3 is empty. When you pour the liquids into beaker 3, the level of the combined liquids corresponds exactly to one of the markings on the side of beaker 3. Which of the following beakers is beaker 3?

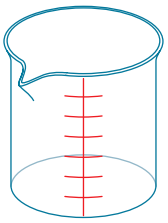
A



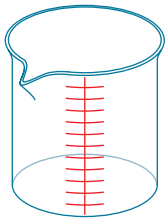
B



C



D



21. The table shows the fraction of the total number of students at Maple Leaf Elementary School that are in each grade.

Kindergarten	$\frac{7}{40}$
Grade 1	$\frac{3}{20}$
Grade 2	$\frac{11}{72}$
Grade 3	$\frac{5}{36}$
Grade 4	$\frac{26}{180}$
Grade 5	$\frac{17}{180}$
Grade 6	$\frac{13}{90}$

a) Which grade has the greatest number of students?

b) Which grade has the least number of students?

c) Which two grades have the same number of students?

d) If there are 54 students in grade 1, what is the total number of students in the school?

MATH LINK

a) Determine a common denominator for the fractions in the Eye of Horus. Show your work.

b) Use this common denominator to determine an equivalent fraction for each part in the eye.