

6.1

Divisibility

Focus on...

After this lesson, you will be able to...


- determine if a number can be divided evenly by 2, 3, 4, 5, 6, 8, 9, 10
- show why a number is not divisible by 0
- find the factors of a number using divisibility rules
- write a fraction in lowest terms using common factors



It's the first day of summer camp. The campers have been divided into 9 groups. Stacy, the camp leader, has a box of 207 "Fun Times Nature Camp" T-shirts. In her head, Stacy quickly figures out that she will be able to divide the 207 T-shirts equally among the 9 groups. How did she do this?

Explore the Math

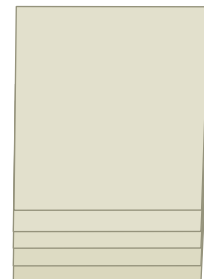
Materials

- number charts 
- coloured pencils
- counters or coins

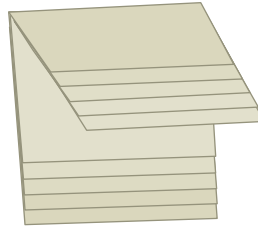


What are the divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10?

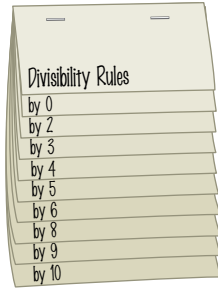
1. Make the following Foldable to organize what you learn in this Explore the Math.
 - a) Use five sheets of paper. Put them in a pile so they overlap by 1.5 cm. Keep the edges straight.



- b) Fold the top edge of the paper.
Stop 1.5 cm from the bottom edge.
Staple together along the fold.



- c) Label the tabs.



Part 1: Divisibility Rules for 2, 5, and 10

2. Use a chart of the numbers 21 to 120.
- Colour each number that is **divisible** by 2 yellow.
 - Circle each number that is divisible by 5.
 - Put an X through each number that is divisible by 10.

21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

3. a) The yellow numbers are divisible by 2. Look at the last digit of each yellow number. Are these digits even or odd?
- b) The circled numbers are divisible by 5. Look at the last digit of each circled number. What do you notice?
- c) The numbers with an X are divisible by 10. Look at the last digit of each number with an X. What do you notice?
- d) Look at the numbers divisible by 10. What other numbers are they divisible by?

divisible

- when a number can be divided by another number, with no remainder

Literacy Link

Even and Odd

Even numbers are 0, 2, 4, 6, 8, and so on.
Odd numbers are 1, 3, 5, 7, 9, and so on.

Reflect on Your Findings

- Describe a divisibility rule for 2.
 - Describe a divisibility rule for 5.
 - Describe a divisibility rule for 10.
 - Add your rules to the Foldable you created in #1. Include examples.

Part 2: Divisibility Rules for 4 and 8

5. Use a chart of the numbers 1044 to 1143.
- Colour each number that is divisible by 4 yellow.
 - Circle each number that is divisible by 8.

1044	1045	1046	1047	1048	1049	1050	1051	1052	1053
1054	1055	1056	1057	1058	1059	1060	1061	1062	1063

For example, the number formed by the last two digits of 1044 is 44.

Literacy Link

Quotient

A quotient is the result of a division. In $12 \div 2 = 6$, the quotient is 6.

- The yellow numbers are all divisible by 4. Look at the last two digits of one of these numbers. What is the number formed by these two digits? Divide it by 2.
 - Is the quotient odd or even? If it is even, divide by 2 again.
 - Is the quotient a whole number or a decimal number?
 - Choose another yellow number. Divide by 2 twice. Is the final quotient a whole number or a decimal number?
 - Choose a number that is not yellow. Divide by 2 twice. Is the final quotient a whole number or a decimal number?
- The circled numbers are divisible by 8. Choose one circled number. Divide it by 2.
 - Is the quotient odd or even? If it is even, divide by 2 again.
 - Is the quotient odd or even? If it is even, divide by 2 again.
 - Is the quotient a whole number or a decimal number?
 - Choose another circled number. Divide by 2 three times. Is the final quotient a whole number or a decimal number?
 - Choose a number that is not circled. Divide by 2 three times. Is the final quotient a whole number or a decimal number?

Reflect on Your Findings

- Describe a divisibility rule for 4.
 - Describe a divisibility rule for 8.
 - Add your rules to the Foldable you created in #1. Include examples.

Part 3: Divisibility Rules for 3, 6, and 9

9. Use a chart of the numbers 0 to 99.
- Colour each number that is divisible by 3 yellow.
 - Circle each number that is divisible by 9.
 - Put an X through each number that is divisible by 6.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19

10. a) The yellow numbers are divisible by 3. Calculate the sum of the digits of a few of these numbers. Continue to calculate the sum of the digits until you get a one-digit number. What number other than 1 is each sum divisible by?
- b) The circled numbers are divisible by 9. Calculate the sum of the digits of a few of these numbers. What is the largest single-digit number that each sum is divisible by? Look at the chart. What other number are these numbers divisible by, other than 1?
- c) Put a vertical blue line | through the numbers that are divisible by 2. The numbers with an X are divisible by 6. What other two numbers are the numbers with an X divisible by?

For example, this is what you do for 93:
 $9 + 3 = 12$
 $1 + 2 = 3$

Reflect on Your Findings

11. a) Describe a divisibility rule for 3.
 b) Describe a divisibility rule for 6.
 c) Describe a divisibility rule for 9.
 d) Add your rules to the Foldable you created in #1. Include examples.

Part 4: Divisibility by 0

12. Use six counters or coins.
- a) Divide your counters into groups of 3. How many groups do you get?
 b) Divide your counters into groups of 2. How many groups do you get?
 c) Divide your counters into groups of 1. How many groups do you get?
 d) Divide your counters into groups of 0. Can you describe how many groups you get? Explain.

13. Copy and fill in the blanks.

- a) $3 \times \blacksquare = 12$ $12 \div 3 = \blacksquare$
 b) $2 \times \blacksquare = 12$ $12 \div 2 = \blacksquare$
 c) $1 \times \blacksquare = 12$ $12 \div 1 = \blacksquare$
 d) $0 \times \blacksquare = 12$ $12 \div 0 = \blacksquare$

Reflect on Your Findings

14. Describe what you learned about divisibility by 0.

Strategies

Look for a Pattern
 Refer to page xvii.

WWW Web Link

For more information about the number zero, go to www.mathlinks7.ca and follow the links.

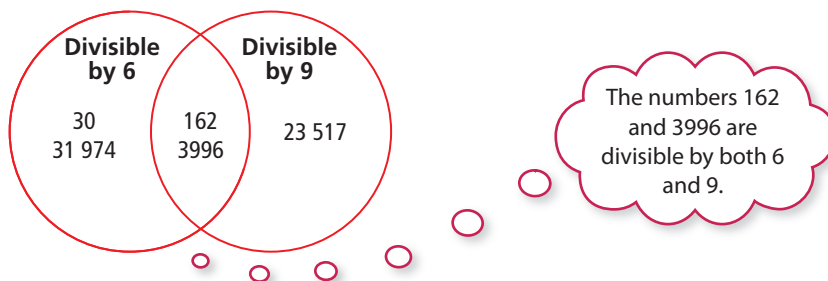
Example 1: Use Divisibility Rules to Sort Numbers

- a) Sort the numbers according to divisibility by 6 and 9.
30 79 162 3996 23 517 31 974
- b) If a number is divisible by both 6 and 9, what is the smallest number other than 1 that it is also divisible by? How do you know?

Solution

- a) Check for divisibility by 6. Is the number divisible by both 2 and 3? Check for divisibility by 9. Is the sum of the digits divisible by 9? Use an organizer such as a Carroll diagram or Venn diagram.

	Divisible by 9	Not Divisible by 9
Divisible by 6	162 3996	30 31 974
Not Divisible by 6	23 517	79



- b) Since 6 is divisible by 3, and 9 is divisible by 3, any number divisible by both 6 and 9 will also be divisible by 3.

The numbers 162 and 3996 are both divisible by 6 and 9, so they will also be divisible by 3.

Check:

$$162 \div 3 = 54$$

$$3996 \div 3 = 1332$$

Show You Know

- a) Sort the numbers according to divisibility by 4 and 5.
93 540 955 8060 67 982 84 430
- b) If a number is divisible by 4 and 5, what other number is it divisible by? How do you know?

Literacy Link

Carroll Diagram

A Carroll diagram is a table that shows how numbers are the same and different.

Literacy Link

Venn Diagram

A Venn diagram shows relationships between groups of numbers.

Example 2: Use Divisibility Rules to Determine Factors

- What are the factors of 24?
- What are the factors of 32?
- What are the **common factors** of 24 and 32?
- What is the greatest common factor of 24 and 32?

Solution

- a) Use divisibility rules to determine the factors.

24 is divisible by 1.

24 is divisible by 2 because it is even.

24 is divisible by 3 because the sum of the digits, $2 + 4 = 6$, is divisible by 3.

24 is divisible by 4 because the number formed by the two digits is divisible by 2 at least twice.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

$$1 \times 24 = 24$$

$$2 \times 12 = 24$$

$$3 \times 8 = 24$$

$$4 \times 6 = 24$$

- b) Use divisibility rules to determine the factors.

32 is divisible by 1.

32 is divisible by 2 because it is even.

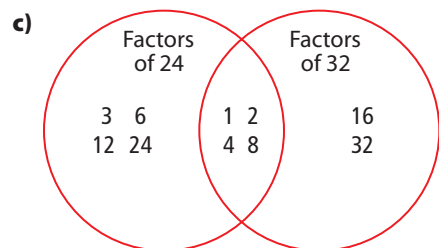
32 is divisible by 4 because the number formed by the two digits is divisible by 2 at least twice.

The factors of 32 are 1, 2, 4, 8, 16, and 32.

$$1 \times 32 = 32$$

$$2 \times 16 = 32$$

$$4 \times 8 = 32$$



The common factors of 24 and 32 are 1, 2, 4, and 8.

- d) The greatest common factor of 24 and 32 is 8.

Show You Know

- What are the factors of 30?
- What are the factors of 18?
- What are the common factors of 30 and 18?
- What is the greatest common factor of 30 and 18?

common factor

- a number that two or more numbers are divisible by
- 4 is a common factor of 8 and 12

All numbers are divisible by 1.

Literacy Link

The greatest common factor is the largest number that both numbers are divisible by.

WWW Web Link

To learn more about factoring and Venn diagrams, go to www.mathlinks7.ca and follow the links.

Example 3: Use Divisibility Rules to Write a Fraction in Lowest Terms

lowest terms

- a fraction is in lowest terms when the numerator and denominator of the fraction have no common factors other than 1

Write $\frac{12}{42}$ in lowest terms.

Solution

Method 1: Divide by Common Factors

Use divisibility rules to determine common factors.

12 is divisible by 2 because it is even.

42 is divisible by 2 because it is even.

Divide the numerator and denominator by the common factor 2.

$$\begin{array}{c} \div 2 \\ \frac{12}{42} = \frac{6}{21} \\ \div 2 \end{array}$$

Keep dividing by common factors until the only common factor is 1. Can you divide again?

6 is divisible by 3.

21 is divisible by 3 because the sum of the digits, $2 + 1 = 3$, is divisible by 3.

Divide by the common factor 3.

$$\begin{array}{c} \div 3 \\ \frac{6}{21} = \frac{2}{7} \\ \div 3 \end{array}$$

Can you divide again?

Stop dividing when the only common factor is 1.

There are no common factors for 2 and 7 other than 1.

$$\frac{12}{42} = \frac{2}{7}$$

So, $\frac{12}{42}$ can be written in lowest terms as $\frac{2}{7}$.

Method 2: Divide by the Greatest Common Factor

Use divisibility rules to determine the factors of 12:

12 is divisible by 1.

12 is divisible by 2 because it is even.

12 is divisible by 3 because the sum of the digits,
 $1 + 2 = 3$, is divisible by 3.

The factors of 12 are 1, 2, 3, 4, 6, and 12.

$$1 \times 12 = 12$$

$$2 \times 6 = 12$$

$$3 \times 4 = 12$$

You do not need to divide by 4, 6, and 12 because the multiplication shows that they are factors of 12.

Use divisibility rules to determine the factors of 42:

42 is divisible by 1.

42 is divisible by 2 because it is even.

42 is divisible by 3 because the sum of the digits,
 $4 + 2 = 6$, is divisible by 3.

42 is divisible by 6 because it is divisible by 2 and 3.

The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

$$1 \times 42 = 42$$

$$2 \times 21 = 42$$

$$3 \times 14 = 42$$

$$6 \times 7 = 42$$

The common factors of 12 and 42 are 1, 2, 3, and 6.

The greatest common factor is 6.

Write $\frac{12}{42}$ in lowest terms by dividing the numerator and denominator by 6.

$$\frac{12}{42} = \frac{2}{7}$$

Diagram showing the simplification of $\frac{12}{42}$ to $\frac{2}{7}$ by dividing both numerator and denominator by 6. Arrows point from 12 to 2 and from 42 to 7, with $\div 6$ written above and below the arrows.

So, $\frac{12}{42}$ can be written in lowest terms as $\frac{2}{7}$.

Show You Know

Write each fraction in lowest terms.

a) $\frac{20}{24}$

b) $\frac{12}{18}$

Key Ideas

Divisibility Rules

A number is divisible by ...	If ...
2	the last digit is even (0, 2, 4, 6, or 8)
3	the sum of the digits is divisible by 3
4	the number formed by the last two digits is divisible by 2 at least twice
5	the last digit is 0 or 5
6	the number is divisible by both 2 and 3
8	the number is divisible by 2 at least three times
9	the sum of the digits is divisible by 9
10	the last digit is 0

- Numbers cannot be divided by 0.
- You can use the divisibility rules to find factors of a number.
- You can write fractions in lowest terms by dividing the numerator and the denominator by common factors until the only common factor is 1.

Communicate the Ideas

- a) Why is a number that is divisible by 6 also divisible by 2 and 3?
 - b) A number is divisible by 10. What other numbers is the number divisible by? How do you know?
- a) Explain one method for determining the greatest common factor of 36 and 20.
 - b) Share your answer with a partner.
3. Simone wrote $\frac{18}{30}$ in lowest terms as $\frac{6}{10}$.

 - a) Is she finished yet? Explain.
 - b) Show a method for writing $\frac{18}{30}$ in lowest terms.
4. Explain what you know about divisibility by 0. Include an example in your explanation.

Practise

For help with #5 to #8, refer to Example 1 on page 202.

5. Which of the following numbers are divisible by 5? Explain how you know.

1010 554 605 902 900 325

6. Which of the following numbers are divisible by 4? Explain how you know.

124 330 3048 678 982 1432

7. a) Use a diagram or table to sort the numbers according to divisibility by 4 and 8.

312 330 148 164 264 13824

- b) If a number is divisible by 4 and 8, what is the smallest number other than 1 that it is also divisible by? How do you know?

8. a) Using a diagram or table, sort the numbers based on divisibility by 6 and 10.

5832 35010 243 9810 31990

- b) If a number is divisible by 6 and 10, what is the smallest number other than 1 that it is also divisible by? How do you know?

For help with #9 to #14, refer to Example 2 on page 203.

9. Use the divisibility rules to list the factors of the following numbers.

a) 36 b) 15 c) 28

10. What are the factors of these numbers?

a) 18 b) 54 c) 72

11. Use the divisibility rules to determine the common factors for each pair of numbers.

a) 3 and 6
b) 4 and 8
c) 6 and 12

12. What are the common factors for each pair of numbers?

a) 5 and 10
b) 4 and 12
c) 24 and 15

13. a) Use the divisibility rules to determine the common factors of 16 and 20. Include a Venn diagram as part of your answer.

b) What is the greatest common factor of 16 and 20?

14. a) What are the common factors of 10 and 30? Include a Venn diagram with your answer.

b) Identify the greatest common factor of 10 and 30.

For help with #15 and #16, refer to Example 3 on pages 204–205.

15. Write the following fractions in lowest terms.

a) $\frac{15}{20}$ b) $\frac{6}{18}$ c) $\frac{10}{16}$
d) $\frac{9}{12}$ e) $\frac{4}{10}$ f) $\frac{9}{15}$

16. Write each fraction in lowest terms.

a) $\frac{12}{16}$ b) $\frac{6}{12}$ c) $\frac{8}{20}$
d) $\frac{14}{24}$ e) $\frac{5}{10}$ f) $\frac{12}{15}$

Apply

17. A shipment of flowers has arrived at Mr. Greenthumb's nursery. He has to sort them into groups.

Flower	Number in Shipment
A daisies	336
B roses	120
C pansies	244
D marigolds	118
E lilies	321

- a) Which flowers can he divide into groups of 2?
- b) Which flowers can he divide into groups of 3?
- c) What is a quick way for Mr. Greenthumb to know which flowers he can divide into groups of 6? Explain.
18. a) Write a five-digit number that is divisible by 3 and 5.
b) Write a seven-digit number that is divisible by 6.
19. A grocery store sells apples in bags of 8 only. Using divisibility rules, determine if you can buy exactly
- a) 116 apples
b) 168 apples
c) 194 apples
20. Anita says that if 6, 10, and 15 are factors of a number, that means 2, 3, and 5 are also factors. Is she correct? Explain how you know.
21. Matthew finds the divisibility rule for 9 difficult to use. Is there a way of making this rule easier to use? Explain.

22. Amouyuk's class and Iblauk's class are going on a trip. There are 30 students in Amouyuk's class and 24 in Iblauk's class. There will be an equal number of students from Amouyuk's class and an equal number from Iblauk's class in each komatik, and no more than 10 students in each.



Komatik, or Sled

- a) How many students will travel in each komatik?
- b) How many komatiks will be needed?
23. There were 12 ripe peaches on a tree. Four children shared them equally. When 12 more peaches were ripe, no children came to pick them. Can the peaches be shared among 0 children? Use this example to explain divisibility by 0.
24. Andrea is the head of the local baseball league. She plans to divide the bats and balls equally among as many teams as she can. There are 16 bats and 40 balls. What is the greatest number of teams she can divide them among? Use a diagram or a table to help you determine the answer.



Extend

25. Adam and Kayla are going to fill their little sister's pool. It holds 616 L of water.
- Which of the following containers can they use to fill the pool exactly to 616 L without having any water left over?
 - 9-L wheelbarrow
 - 8-L barrel
 - 6-L bucket
 - 2-L jug and a 5-L pail
 - For the containers they can use, how many of each container would they need?



26. A parallelogram has an area of 48 cm^2 and a rectangle has an area of 64 cm^2 . They have the same base.
- What is the least possible height the parallelogram could have?
 - What is the least possible height the rectangle could have?
27. George is arranging sandwiches on a tray for a class party. If he arranges the sandwiches in rows of 2, 3, 4, 5, or 6 he always has exactly one sandwich left over each time. What is the smallest number of sandwiches that he could have?



28. What is the smallest number you must add to each of the following numbers so that the sum is divisible by 3?
- $24683 + \blacksquare$
 - $502 + \blacksquare$
 - $5439 + \blacksquare$

MATH LINK

The diagram shows the fraction of time Joseph spends on all his activities during a 24-h day.

- Use the divisibility rules to find the factors of 24.
- Use the factors of 24 to help you write the fractions in the diagram in lowest terms.
- Are there fractions that you could not rewrite in lowest terms? Which ones? Why?
- How would you change the diagram now that you have written the fractions in lowest terms? Explain.

